

Soft wall model with inverse exponential profile as a model for the axial and pseudoscalar mesons

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Abstract

According to the conjecture of the gauge/gravity duality a strongly coupled 4D field theory may be described by a dual 5D theory which is a free field theory in the first approximation. The usual description of the axial-vector sector in the bottom-up AdS/QCD models includes, however, the quartic interaction with a bulk scalar field. The question appears whether it is possible to describe some aspects of the chiral symmetry breaking in a way more consistent with the original proposal, *i.e.* within a free 5D theory? We suggest that a natural candidate for this purpose is the soft-wall model with inverse exponential profile.

1 Introduction

The ideas of AdS/CFT correspondence from the string theory [1–3] have inspired the appearance of holographic approach to QCD, in particular of the bottom-up holographic models. These models try to describe the non-perturbative dynamics of strong interactions in terms of a putative semiclassical five-dimensional theory. The first versions of bottom-up models describing the meson spectrum and the Chiral Symmetry Breaking (CSB) — the so-called hard wall models [4, 5] — had incorrect predictions for the behavior of spectrum of radial meson excitations ($m_n \sim n$ instead of the string like behavior $m_n^2 \sim n$ expected in the phenomenology [6]) and for the non-perturbative corrections to the leading parton logarithm of two-point correlation functions (exponential instead of expected polynomial corrections). The introduction of the Soft Wall (SW) holographic model [7] has solved these problems.

The situation with a self-consistent description of the CSB is more complicated. To reflect the Goldstone theorem and related phenomenology like the mass splitting between the vector and axial-vector states one introduces

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the interaction term(s) in a 5D holographic action. The resulting models are at odds with the initial assumption that non-perturbative dynamics of a strongly coupled 4D gauge theory, in the first approximation, admits a dual description in terms of a free 5D field theory. Let alone a sad fact that following this way one abandons the hope to learn something new about the strongly coupled QCD from the holographic approach. Instead of this one attempts to find a 5D description of old known physics that is not worse than the descriptions given by the old known effective theories for QCD.

Within the usual effective models describing the strong interactions at low energies, the dynamics responsible for the CSB is separated from the dynamics responsible for the confinement. The latter phenomenon is not necessary present in the effective quark models (for instance, it is absent in the Nambu–Jona-Lasinio like models). *A priori* it is not evident why the separation of the CSB from the confinement can be done in the 5D dual theories. Since the holographic models yield colorless mesons at any energies the confinement is definitely present by construction. It is not excluded, however, that a part of the CSB physics is encoded in the choice of 5D background. If this is the case then by a successful choice of 5D background we can catch some important aspects of the CSB on the level of a free holographic theory.

In building the holographic models for QCD it should be always kept in mind that the conjectural outcome of these models is given by the correlators of currents in the large- N_c limit. They encode various dynamical information. The spectrum is a part of this information. The equations of motion provide just the alternative way to find the spectrum without calculating the two-point correlators. The correlators represent thus the primary objects which must be compared with their QCD counterparts. It is known that the axial-vector two-point correlator should contain a massless (in the chiral limit) pion pole. This is an important consequence of the CSB. Basing on this observation we suggest that the free action of the SW model with inverse dilaton is a natural candidate for the description of the axial-vector channel, with the CSB taken into account in the first approximation. In the present work, we elaborate this point.

The paper is organized as follows. The basics of the SW models are reminded in the next section, where we discuss the advantages and disadvantages of these models. In Sect. 3, it is shown that the SW model with inverse dilaton is a good first approximation for the axial channel. The improvement of the SW model with inverse dilaton is presented in Sect. 4. The Sect. 5 contains a brief discussion of the scalar sector. Then, in Sect. 6, we make some remarks and conclude in Sect. 7.

2 The soft wall model

In this section, we briefly summarize the main results of the holographic SW model. The simplest action of the SW model describing the vector mesons is given by

$$S = -\frac{c^2}{4} \int d^4x dz \sqrt{g} e^{-az^2} F_{MN} F^{MN}, \quad (1)$$

where $F_{MN} = \partial_M V_N - \partial_N V_M$, $M = 0, 1, 2, 3, 4$ (the metric is mostly negative), and c represents a normalization constant for the field V_M . The action (1) is defined in the AdS_5 space whose metric can be parametrized as follows,

$$ds^2 = \frac{R^2}{z^2} (dx_\mu dx^\mu - dz^2), \quad z > 0, \quad (2)$$

here R is the radius of the AdS_5 space and z is the holographic coordinate. On the boundary of the AdS_5 space, $z = \epsilon \rightarrow 0$, the field V_M corresponds to the source for a QCD operator interpolating the vector mesons, $V_M(x, \epsilon) \leftrightarrow \bar{q} \gamma_\mu q$ or $V_M(x, \epsilon) \leftrightarrow \bar{q} \gamma_\mu \gamma_5 q$. According to the prescriptions of the AdS/CFT correspondence [2, 3], the masses of fields in a dual theory defined on the AdS_5 space are

$$m_5^2 R^2 = (\Delta - J)(\Delta + J - 4), \quad (3)$$

where Δ is the canonical dimension of the corresponding 4D field theory operator and J is the spin, $J = 0, 1$. We will set $R = 1$ in what follows. In the case under consideration, $\Delta = 3$. This canonical dimension for the vector current results in the zero mass for the 5D vector field V_M .

Recently some questions appeared concerning the choice of the sign of the exponential profile (called also dilaton profile) presenting in the action of the SW models. In particular, it was proposed that the inverse (with respect to the choice of Ref. [7]) dilaton profile provides nicer confinement properties [8, 9] and better description of the CSB [9]. However, the SW model with inverse dilaton leads to the appearance of massless pole in the vector correlator which cannot be eliminated [10].

The action (1) is purely phenomenological. It is not known which 5D dynamical model leads to the particular background e^{-az^2} as a solution of 5D Einstein equations. An "intermediate" dynamical model that leads to such a background could look as follows,

$$S = \int d^4x dz \sqrt{g} (\partial_M \varphi \partial^M \varphi - m^2 \varphi^2 + e^\varphi \mathcal{L}), \quad (4)$$

where the 5D space is the AdS one and \mathcal{L} represents some Lagrangian density. If the 5D mass squared m^2 of the "dilaton" field φ takes the minimal value

permitted in the AdS space, $m^2 = -4$ (the Breitenlohner-Freedman stability bound [11]), then the equation of motion (see (34)) will have a solution $\varphi = -az^2$. The sign of the constant a is not fixed. In fact, it is difficult to give dynamical arguments which would allow to fix this sign. On the heuristic level, since z is the inverse energy scale one may imagine that the background e^{-az^2} acts as a "projector": The choice $a < 0$ enhances the part of the action that is important in the infrared limit. Thus, the scalar part should contain the pseudoscalar mesons and the vector part should contain the axial-vector field because the pions emerge from the divergence of the axial-vector current. The same heuristic argument suggests the choice $a > 0$ for the description of the scalar and vector states.

The hypothesis of AdS/CFT correspondence yields a practical tool for calculation of correlation functions in a strongly coupled 4D field theory [2,3]. All such functions can be obtained from the generating functional of the connected correlators $W_{4D}[\varphi_0(x)]$ that depends on the sources $\varphi_0(x)$ for the 4D field theory operators. If the 5D dual theory exists, the holographic correspondence postulates the identification

$$W_{4D}[\varphi_0(x)] = S_{5D}[\varphi(x, \epsilon)]. \quad (5)$$

The 5D dual theory is assumed to be in the weakly coupled regime. This implies two important consequences: As the first approximation, we may restrict ourselves by the quadratic terms in S_{5D} and use the semiclassical analysis. Thus, roughly speaking, the recipe for calculation of n-point correlators is as follows: Evaluate S_{5D} on the solution of equation of motion and differentiate n times with respect to boundary values of 5D fields.

Let us apply this recipe to the action (1). This action is gauge invariant so we may use the gauge freedom to fix the axial gauge, $V_z = 0$, which is the most convenient. In addition, we will consider the transverse fields, $\partial_\mu V^\mu = 0$. The dependence on the usual 4D coordinates of field $V^\mu(x, z)$ can be Fourier transformed to $V^\mu(q, z)$. If we let $V^\mu(q, z) = v(q, z)V_0^\mu(q)$ and impose $v(q, \epsilon) = 1$ then $V_0^\mu(q)$ can be interpreted as the source for the operator of vector current. The equation of motion for the scalar function $v(q, z)$ follows from the action (1),

$$\partial_z \left(\frac{e^{-az^2}}{z} \partial_z v \right) + \frac{e^{-az^2}}{z} q^2 v = 0. \quad (6)$$

Evaluation of the action (1) on the solution of Eq. (6) yields the boundary term

$$S = \frac{c^2}{2} \int d^4x V_0^\mu V_{0\mu} \frac{e^{-az^2}}{z} v \partial_z v \Big|_{z=\epsilon}^{z=\infty}. \quad (7)$$

The two-point correlator $\Pi_V(-q^2)$ of vector currents J_μ is defined as

$$\int d^4x e^{iqx} \langle J_\mu(x) J_\nu(0) \rangle = (q_\mu q_\nu - q^2 g_{\mu\nu}) \Pi_V(-q^2). \quad (8)$$

It can be found by differentiating twice with respect to source V_0^μ in (7) near the boundary $z = \epsilon$,

$$\Pi_V(-q^2) = c^2 \left. \frac{\partial_z v}{q^2 z} \right|_{z=\epsilon}. \quad (9)$$

Solution of Eq. (6) with the boundary conditions $v(q, \epsilon) = 1$, $v(q, \infty) = 0$ is

$$v(q, z) = \Gamma \left(1 - \frac{q^2}{4|a|} \right) e^{(a-|a|)z^2/2} U \left(\frac{-q^2}{4|a|}, 0; |a|z^2 \right), \quad (10)$$

where U is the Tricomi confluent hypergeometric function. Substitution of (10) in (9) results in

$$\Pi_V(-q^2) = c^2 \left[\frac{a - |a|}{q^2} - \frac{1}{2} \psi \left(1 - \frac{q^2}{4|a|} \right) \right] + \text{const.} \quad (11)$$

The digamma function ψ has the following representation

$$\psi \left(1 - \frac{q^2}{4|a|} \right) = \sum_{n=0}^{\infty} \frac{4|a|}{q^2 - 4|a|(n+1)} + \text{const.} \quad (12)$$

Using (12) one arrives at the pole representation for the vector correlator,

$$\Pi_V(-q^2) = c^2 \left[\frac{a - |a|}{q^2} + \sum_{n=0}^{\infty} \frac{2|a|}{4|a|(n+1) - q^2} \right] + \text{const.} \quad (13)$$

The poles in the sum of expression (13) give the mass spectrum of the model².

As is seen from the expression (13), the sign choice for a is important. If the sign choice is negative, $a < 0$, the vector correlator contains the massless pole. The physical origin of this pole has been nicely discussed in Ref. [10]. Rephrasing the essence in a short way, the vector physical modes are looked for in the form

$$V_\mu(q, z) = \varepsilon_\mu e^{iqx} v(z), \quad (14)$$

²The independence of spectrum of massive states on the sign of a is a peculiarity of the vector channel. For instance, a straightforward calculation shows that the spectrum of the scalar mesons described by the quadratic in fields 5D action does not have this property (see Eq. (39)).

where ε_μ denotes the polarization vector, e^{iqx} is the 4D plane wave, and $v(z)$ represents a profile depending on the holographic coordinate. The spectrum of massive modes corresponds to normalizable solutions of Eq. (6) whose eigenvalues yield $q_n^2 = m_n^2$. These solutions satisfy the boundary condition $v(\epsilon) = 0$ and leave the action (7) finite. However, there is a massless non-normalizable solution $v(z) = e^{az^2}$ which also leaves the action (7) finite if $a < 0$.

Note that at large euclidean momentum $Q^2 = -q^2$ the correlator (11) has the following asymptotic expansion

$$\Pi_V(Q^2)_{Q^2 \rightarrow \infty} = \frac{c^2}{2} \left[\log \left(\frac{4|a|}{Q^2} \right) - \frac{2a}{Q^2} + \frac{4a^2}{3Q^4} + \mathcal{O} \left(\frac{a^4}{Q^8} \right) \right]. \quad (15)$$

On the other hand, in QCD sum rules, the Operator Product Expansion (OPE) for the same correlator reads [12],

$$\Pi_V(Q^2)_{\text{OPE}} = \frac{N_c}{24\pi^2} \log \left(\frac{\mu^2}{Q^2} \right) + \frac{\alpha_s}{24\pi} \frac{\langle G^2 \rangle}{Q^4} + \xi \frac{\langle \bar{q}q \rangle^2}{Q^6} + \mathcal{O} \left(\frac{\mu^8}{Q^8} \right), \quad (16)$$

where $\langle G^2 \rangle$ and $\langle \bar{q}q \rangle$ mean the gluon and quark condensate, μ is a renormalization scale, and the constant ξ is different for the vector and axial-vector case. Comparison of expressions (15) and (16) leads to the following conclusions: a) the normalization factor c for the 5D vector field is fixed,

$$c^2 = \frac{N_c}{12\pi^2}; \quad (17)$$

b) the sign of $\langle G^2 \rangle$ is reproduced correctly; c) the $\mathcal{O}(Q^{-6})$ contribution is absent as we expected since the effect of quark condensate was not included into the model; d) the presence of $\mathcal{O}(Q^{-2})$ contribution disagrees with QCD as long as its numerator would correspond to a local gauge-invariant condensate of dimension two which cannot be constructed in QCD. The last point represents a serious drawback of the SW models that seems to be ignored in the literature³.

3 The model with inverse dilaton corresponds to the axial mesons

The vector spectrum of the SW model is independent of the sign of slope parameter a . However, the choice $a < 0$ leads to the massless state in

³Except the Ref. [13] where this observation was the main motivation to modify the SW model.

the vector channel that contradicts QCD. For this reason the model with $a < 0$ was discarded in the original paper [7], where the SW model was introduced. On the other hand, it is known that the axial-vector channel does contain the massless (in the chiral limit) pseudoscalar state due to the Partial Conservation of Axial Current (PCAC) hypothesis. Consequently, the model with inverse dilaton, $a < 0$, may be interpreted as a natural SW model for the axial-vector mesons. We wish to present some additional arguments in favor of our conjecture.

First of all, the spectrum of the model behaves as $m^2 \sim n + 1$. Theoretically, such a spectrum is typical for the axial-vector mesons while the vector mesons should have $m^2 \sim n + 1/2$ with the same slope. This pattern of spectrum holds in the generalized Lovelace-Shapiro dual amplitude [14] which was phenomenologically the most successful among the dual amplitudes of the Veneziano type. Also it appeared in the early versions of QCD planar (= large- N_c) sum rules [15]. The comparison with the latter method is crucial. Concerning the problems of meson spectroscopy, the bottom-up holographic models may be interpreted as an exact five-dimensional reformulation of QCD planar sum rules [16]. Choosing the negative sign for the parameter a we incorporate an important feature of the chiral symmetry breaking in QCD — the PCAC. As a result, the residue in the massless pole must be interpreted as f_π^2 , where $f_\pi = 92.4$ MeV is the pion decay constant. But we do not yet incorporate another order parameter of CSB — the quark condensate. In the original SVZ sum rules [12], the latter had impact on the masses of mesons on the level of several percent only. Thus, except the pseudoscalar states, discussing the spectrum of mesons in the large- N_c limit of QCD, it seems to be a good first approximation to neglect the quark condensate. Saturating the two-point correlators by the spectrum linear in masses squared, the planar QCD sum rules can be solved in this approximation, the result for the vector (V) and axial-vector (A) correlators is [17] (possible corrections to this picture are considered in Ref. [18])

$$\Pi_V(Q^2) = \sum_{n=0}^{\infty} \frac{2f_\pi^2}{Q^2 + \Lambda(n + 1/2)} + \text{const}, \quad (18)$$

$$\Pi_A(Q^2) = \frac{f_\pi^2}{Q^2} + \sum_{n=0}^{\infty} \frac{2f_\pi^2}{Q^2 + \Lambda(n + 1)} + \text{const}, \quad (19)$$

where

$$\Lambda = \frac{48\pi^2}{N_c} f_\pi^2. \quad (20)$$

The spectrum has the pattern mentioned above and is parametrized completely by the constant f_π . The result (20) follows for our slope $4|a|$ from

the comparison of residues of massive states in (13) and (19) taking into account the normalization factor (17). However, the massless residue in the correlator (13) for $a < 0$ turns out to be twice the massless residue in the axial correlator (19).

Thus, interpreting the SW model with $a < 0$ as a model for the axial-vector mesons we encounter two problems: (1) the presence of dimension-two condensate; (2) the value of pion residue is too large. Both problems disappear if we reformulate the SW model as a model without the dilaton profile. This model will be considered in the next section.

From the phenomenological point of view, the spectrum of the SW model,

$$m_n^2 = m_0^2(n+1), \quad n = 0, 1, 2, \dots \quad (21)$$

also corresponds to the axial-vector mesons rather than to the vector states. For comparison of the spectrum (21) with the experimental data it is convenient to display it in units of mass squared of the ground state,

$$m_n^2 = m_0^2\{1, 2, 3, 4, \dots\}. \quad (22)$$

According to the Particle Data [19], the spectrum of ρ -mesons is (in MeV, experimental errors are neglected): 775, 1465, 1720, 1900^[?], 2000^[??], 2149^[?], 2265^[??], where the sign [?] marks the states which need confirmation and the sign [??] marks badly established resonances which were usually seen by one collaboration only. Using the form (22) this data can be displayed as follows,

$$m_{\rho,n}^2 = m_\rho^2\{1, 3.6, 4.9, 6.0^{[?]}, 6.7^{[??]}, 7.7^{[?]}, 8.5^{[??]}\}. \quad (23)$$

Comparison of (23) with (22) does not reveal similarity.

Consider the spectrum of known axial-vector states a_1 [19]: 1230, 1640^[?], 1930^[??], 2265^[??]. In the form of (22), it reads

$$m_{a_1,n}^2 = m_{a_1}^2\{1, 1.8^{[?]}, 2.5^{[??]}, 3.4^{[??]}\}. \quad (24)$$

The spectrum (24) resembles (22), at least qualitatively.

A better quantitative agreement with the axial-vector spectrum is achieved if one introduces the ultraviolet cutoff $z = \Lambda_{\text{cut}}$ to the SW model. The introduction of Λ_{cut} looks reasonable as long as QCD is weakly coupled in the ultraviolet domain, hence, its hypothetical holographic dual theory should be strongly coupled in that domain and the semiclassical treatment with the latter is not justified any more. The vector correlator of the SW model with the ultraviolet cutoff was calculated in Ref. [20],

$$\Pi_V(-q^2) = c^2 \frac{e^{-|a|\Lambda_{\text{cut}}^2}}{2} \frac{U(1 - q^2/(4|a|), 1, |a|\Lambda_{\text{cut}}^2)}{U(-q^2/(4|a|), 0, |a|\Lambda_{\text{cut}}^2)}. \quad (25)$$

The poles of correlator (25) yield the mass spectrum which becomes now nonlinear. In particular, the mass of the first excited state gets closer to the mass of the ground one, $m_1^2/m_0^2 < 2$. This corresponds indeed to the experimental axial-vector spectrum and is contrary to the situation in the vector one. For instance, the choice $|a|\Lambda_{\text{cut}}^2 = 1$ gives rise to the following spectrum,

$$m_n^2 = m_0^2 \{1, 1.8, 2.6, 3.3, 4.1, \dots\}, \quad (26)$$

which agrees very well with the experimental data on the a_1 -mesons (24).

4 Improvement: The no-wall model

In this Section, we indicate a possible way to solve the problems (1) and (2) of the standard SW model. In fact, this way has been already proposed in Ref. [13]. We will repeat the main steps and add some new points which were not mentioned in Ref. [13].

Let us remove the dilaton profile in the action (1) with the help of the transformation [13]

$$V_M = e^{az^2/2} \tilde{V}_M. \quad (27)$$

The action becomes equivalent to (we will not write the normalization constant)

$$S = -\frac{1}{4} \int d^4x dz \sqrt{g} \left(\tilde{F}_{MN} \tilde{F}^{MN} + \frac{a^2 z^4}{2} \tilde{V}_\mu \tilde{V}^\mu \right) + \frac{a}{2} \int d^4x \tilde{V}_\mu^2 \Big|_{z=0}^{z=\infty}. \quad (28)$$

The last surface term was omitted in Ref. [13]. We keep it for the reason that now will be clear. The holographic profile of physical modes (14) has the following behavior

$$v(z) \sim z^k e^{(a-|a|)z^2/2}, \quad k > 0. \quad (29)$$

The surface term in the action (28) disappears if $a < 0$ and is infinite if $a > 0$. Hence, the reformulation of the SW model of the kind (28) is possible only for the case $a < 0$.

As was demonstrated in Ref. [13], the vector correlator of the no-wall model is (we insert again the normalization factor)

$$\Pi_V(-q^2) = c^2 \left[-\frac{|a|}{q^2} + \sum_{n=0}^{\infty} \frac{2|a|}{4|a|(n+1) - q^2} \right] + \text{const}, \quad (30)$$

with the asymptotic expansion at large $Q^2 = -q^2$

$$\Pi_V(Q^2)_{Q^2 \rightarrow \infty} = \frac{c^2}{2} \left[\log \left(\frac{4|a|}{Q^2} \right) + \frac{4a^2}{3Q^4} + \mathcal{O} \left(\frac{a^4}{Q^8} \right) \right]. \quad (31)$$

Now the massless residue is the half of massive residue and the $\mathcal{O}(Q^{-2})$ contribution in the expansion (31) is absent. Thus, both our goals are achieved simultaneously.

The price to pay is the appearance of the z -dependent mass term in the action (28). This term breaks the original gauge invariance. There is an easy trick that restores the gauge invariance: the emerging z -dependent mass may be interpreted as an effect of condensation of some bulk scalar field which is coupled to the vector field via the covariant derivative (we omit tildes henceforth),

$$S = \int d^4x dz \sqrt{g} \left(|D_M \varphi|^2 - m_\varphi^2 \varphi^2 - \frac{1}{4} F_{MN} F^{MN} \right), \quad (32)$$

where

$$D_M = \partial_M - i\lambda V_M. \quad (33)$$

The action (32) contains the quartic interaction which we have tried to avoid. Nevertheless, let us see how this trick works. The equation of motion for φ in the absence of the vector field V_M is

$$-\partial_z \left(\frac{\partial_z \varphi}{z^3} \right) + \frac{m_\varphi^2 \varphi}{z^5} = 0. \quad (34)$$

To give the desired condensate term it must have a solution $\varphi_0 \sim z^2$. This takes place if $m_\varphi^2 = -4$. The naive application of relation (3) results in the conclusion that the field φ corresponds to an operator of canonical dimension $\Delta = 2$. This conclusion is debatable. According to an AdS/CFT prescription [21], the solution of classical equation of motion for a scalar field Φ corresponding to an operator O has the following form near the 4D boundary $z \rightarrow 0$,

$$\Phi(x, z)_{z \rightarrow 0} = z^{4-\Delta} \Phi_0(x) + z^\Delta \frac{\langle O(x) \rangle}{2\Delta - 4}, \quad (35)$$

where $\Phi_0(x)$ acts as a source for $O(x)$ and $\langle O(x) \rangle$ denotes the corresponding condensate. It is seen that at $\Delta = 2$ the relation (35) is not well defined.

The fact that the no-wall model describes the discrete spectrum of axial-vector mesons and not of the vector mesons can be demonstrated straightforwardly. Following [4, 5], let us introduce the left (L) and right (R) 5D vector fields corresponding on the 4D boundary to the sources for the left and right vector currents, $A_L^M(x, \epsilon) \leftrightarrow \bar{q}_L \gamma^\mu q_L$ and $A_R^M(x, \epsilon) \leftrightarrow \bar{q}_R \gamma^\mu q_R$. They are related to the usual V and A fields as $V = A_L + A_R$, $A = A_L - A_R$. Then the action of no-wall model is

$$S = \int d^4x dz \sqrt{g} \left(|D_M \varphi|^2 + 4\varphi^2 - \frac{1}{4} F_L^2 - \frac{1}{4} F_R^2 \right), \quad (36)$$

Since the reflection of coordinate means in 5D space the interchange of left and right fields, the spatial parity dictates the following form for the covariant derivative [4, 5]: $D_M = \partial_M - i\lambda(A_{L,M} - A_{R,M})$. The action takes the form

$$S = \int d^4x dz \sqrt{g} \left(|\partial_M \varphi - i\lambda A_M \varphi|^2 + 4\varphi^2 - \frac{1}{8}F_A^2 - \frac{1}{8}F_V^2 \right). \quad (37)$$

As follows from (37), the spectrum of A -mesons will be discrete, $m_A^2 \sim n+1$, while the spectrum of V -mesons will be continuous.

Obviously, the case of $U_L(1) \times U_R(1)$ gauge symmetry discussed above can be generalized to the $SU_L(2) \times SU_R(2)$ case.

5 The scalar sector

Let us consider the free scalar action of the SW model,

$$S = \int d^4x dz \sqrt{g} e^{-az^2} (\partial_M \Phi \partial^M \Phi - m^2 \Phi^2). \quad (38)$$

The ensuing equation of motion results in the following spectrum,

$$m_n^2 = 2|a| \left(2n + 1 + \sqrt{4 + m_5^2} + \frac{a}{|a|} \right), \quad n = 0, 1, 2, \dots \quad (39)$$

We are interested in the situation when the massless scalar state appears. There is only one possibility: $a < 0$, $m_5^2 = -4$. As we indicated above, the value $m_5^2 = -4$ corresponds to the minimal value for the mass squared in the AdS space [11]. This possibility has been mentioned in Ref. [8].

Following our proposal in Section 2, the choice $a < 0$ should correspond to the axial mesons in the vector channel and to the pseudoscalar mesons in the scalar one. It looks encouraging to observe that with this choice of sign the appearance of the massless pole in the vector correlator may be accompanied by the appearance of the massless state in the scalar part of the action. It is interesting to remark also that the comparison of the ensuing pseudoscalar spectrum

$$m_n^2 = 4|a|n, \quad (40)$$

with the axial spectrum, $m_n^2 = 4|a|(n+1)$, leads to the prediction that $m_{\pi'} = m_{a_1}$, where π' denotes the first radial excitation of the pion. This prediction is compatible with the experiment [19]: $m_{\pi'} = 1300 \pm 100$ MeV, $m_{a_1} = 1230 \pm 40$ MeV.

6 Discussions

As has been noticed in Ref. [10], aside from the existence of massless state in the vector channel, there is an additional argument against the $a < 0$ choice in the SW models: it would give a higher spin meson spectrum independent of spin j , $m_{n,j}^2 = 4|a|(n+1)$, while the SW model with the positive sign $a > 0$ gives the expected string like spectrum $m_{n,j}^2 = 4|a|(n+j)$ [7]. We wish to make a couple of comments on this arguments. First, it does not exclude the proposal of the present work since the negative sign $a < 0$ can be viewed as a peculiarity of the axial-vector channel. Second, this argument seems to be valid for the gauge higher spin fields in AdS_5 which can be made massless by a special choice of the gauge conditions. However, such an introduction of higher spin fields is not unique. For instance, Brodsky and Teramond advocated in the Ref. [22] that the relation (3) for the 5D mass extends to the $J > 1$ mesons. In this case, the argument above is not valid (see, e.g., Ref. [23]).

It should be emphasized that the massless pole in the axial-vector correlator does not imply the existence of massless axial-vector state. This can be seen as follows. The physical states correspond to normalizable solutions of the kind (14). The corresponding eigenfunctions $v_n(z)$ form a complete set of functions. Thus, it is possible to make the expansion $A^\mu(x, z) = \sum_n A_n^\mu(x) v_n(z)$. Substituting this expansion back into the original action and integrating over z one arrives at a 4D action containing the infinite number of free fields with masses squared m_n^2 given by the eigenvalues corresponding to the eigenfunctions $v_n(z)$ (the procedure is written in detail in Ref. [16]),

$$S = c^2 \int d^4x \sum_{n=0}^{\infty} \left[-\frac{1}{4} (F_{\mu\nu}^{(n)})^2 + m_n^2 (A_\mu^{(n)})^2 \right] \quad (41)$$

The massless state does not appear in the 4D action since it is non-normalizable.

We note finally that the existence of a massless non-normalizable solution that leaves the action finite, in the general case, does not lead to the massless pole in the corresponding correlator. For instance, the Eq. (6) has the second massless non-normalizable solution, $v(z) = \text{const}$, which leaves the action (7) finite at any a (namely, giving zero contribution to the action). This fact becomes especially evident if one considers a 5D action for the scalar fields or non-gauge vector fields⁴, where such solutions can yield a non-zero contribution to the action.

⁴Because in contrast to the definition (8), in these cases one does not extract the factor q^2 in defining the two-point correlator.

7 Conclusions

The chiral symmetry breaking is usually introduced into the bottom-up holographic models by hands via an extra scalar field or special boundary conditions. This strategy in the search for successful dynamical model is inherited from the low-energy effective field theories for QCD. As long as the foundations of holographic correspondence are not yet well understood, it is not clear why we may separate the dynamics responsible for the formation of masses from the corrections to this dynamics caused by the chiral symmetry breaking. In fact, building the five-dimensional dual models we cannot *a priori* control such a separation. We have argued in the present work that in the axial-vector sector, the soft wall model with the exponentially growing quadratic background includes a substantial part of the CSB already on the level of free fields⁵. The massless pole in the correlator of conserved vector currents (not corresponding to a real massless vector particle) actually means that the given current is conserved due to the presence of a massless scalar π such that $A_\mu \sim \partial_\mu \pi$. Thus, the PCAC hypothesis becomes a prediction of the model. In addition, the spectrum of the model resembles much more the spectrum of known axial states than the vector one.

Aside from the massless pole in the axial correlator, an interesting unexpected property of changing the dilaton sign is the possibility to have automatically the massless state in the scalar channel. In summary: The standard soft-wall model seems to be good for the description of the vector and scalar mesons while for the axial-vector and pseudoscalar channels it looks better to change the sign of the dilaton background.

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⁵As a consequence, the standard introduction of the CSB via adding the 5D scalar field dual to the quark operator $\bar{q}q$ may meet the double counting problem.

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